

Math 323 - Formal Mathematical Reasoning and Writing  
Problem Session  
Wednesday, 4/8/15

1. Let  $\mathcal{R}$  be a relation on the set  $A$ . Define what it means for the relation  $\mathcal{R}$  to have trichotomy.
2. In each of the following cases, determine if  $\mathcal{R}$  defines an equivalence relation on  $\mathbb{Z}$ . Which of these has trichotomy?
  - (a)  $x\mathcal{R}y$  if and only if  $x \leq y$
  - (b)  $x\mathcal{R}y$  if and only if  $x \neq y$
  - (c)  $x\mathcal{R}y$  if and only if  $xy > 0$
3. Let  $n$  be a fixed natural number greater than 1. Define an equivalence relation  $\sim$  on  $\mathbb{N}$  as follows: if  $a, b \in \mathbb{N}$ , we say that  $a \sim b$  if and only if there exists some  $k \in \mathbb{Z}$  such that  $a = b \cdot n^k$ .

Prove that  $\sim$  is an equivalence relation.

4. Consider the set  $\bigcup_{n \in \mathbb{N}} \left(0, \frac{3n}{n+2}\right)$ . This set is an interval in  $\mathbb{R}$ . Use interval notation to say what  $S$  is, and prove that  $S$  is this interval.

## L<sup>A</sup>T<sub>E</sub>X tip of the week!

We like curly R's for relations. Like:  $\mathcal{R}$ . You can use `\mathcal` or load the package `euscript` with the option `mathscr` in the preamble with: `\usepackage[mathscr]{euscript}` and use `\mathscr`:

- `$$\mathcal{R}$$`:  $\mathcal{R}$
- `$$\mathscr{R}$$`:  $\mathcal{R}$  (requires the package `euscript`)